

The Modified Group Expansions for Construction of Solutions to the BBGKY Hierarchy

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A solution to the BBGKY hierarchy for nonequilibrium distribution functions is obtained within modified boundary conditions. The boundary conditions take into account explicitly both the nonequilibrium one-particle distribution function as well as local conservation laws. As a result, modified group expansions are proposed. On the basis of these expansions, a generalized kinetic equation for hard spheres and a generalized Bogolubov–Lenard–Balescu kinetic equation for a dense electron gas are derived within the polarization approximation.

KEY WORDS: Nonequilibrium distribution function(s); kinetic equation(s); collision integral(s).

1. INTRODUCTION

The construction of kinetic equations for *dense* gases, liquids and plasma, where a small parameter (density or interaction) is absent, still remains one of the most significant complication in the kinetic theory of classical systems. A number of difficult tasks centre around it. The study of solutions to the BBGKY hierarchy for nonequilibrium distribution functions in view of interparticle correlations could be considered as a way to solve this problem.

Recently, a kinetic equation of the revised Enskog theory for a dense system of hard spheres and an Enskog–Landau kinetic equation for a dense system of charged hard spheres have been obtained from the BBGKY hierarchy in the “pair” collisions approximation.^(1–3) It is necessary to

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notice that this approximation does not correspond to the usual two-particle approximation, inherent in the Boltzmann theory, because an essential part of the many-particle correlations is implicitly taken into account by the pair quasiequilibrium distribution function $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$.

To analyze solutions to the BBGKY hierarchy^(1,2) in higher approximations on interparticle correlations, it is more convenient to use the concept of group expansions.⁽⁴⁻⁶⁾ This has been applied to the BBGKY hierarchy in previous investigations⁽⁴⁻¹⁷⁾ using boundary conditions which correspond to the weakening correlations principle by Bogolubov.⁽¹⁸⁾ The same conception has been involved in papers of Zubarev and Novikov,^(5, 19-21) where a diagram method for obtaining solutions to the BBGKY hierarchy was developed. Moreover, in papers by Zubarev *et al.*^(22, 23) a *consistent* description of kinetics and hydrodynamics has been proposed. It is based on a new formulation of boundary conditions for the Liouville equation and BBGKY hierarchy. In particular, this formulation takes into account interparticle correlations which are connected with local conservation laws.

In this paper, the conception of group expansions will be applied to the BBGKY hierarchy with modified boundary conditions which take into account both the nonequilibrium behaviour of one-particle distribution function and local conservation laws (Section 2). In Sections 3 and 4 it will be demonstrated on the basis of the modified group expansions how to obtain a generalization of some known kinetic equations for hard spheres and homogeneous plasma to the case of high densities. Advantages and shortcomings of the proposed approach are discussed concisely in conclusion and at the end of some sections.

2. THE BBGKY HIERARCHY WITH MODIFIED BOUNDARY CONDITIONS

The BBGKY hierarchy of equations for nonequilibrium distribution functions of classical interacting particles has been obtained in the paper⁽²²⁾ on the basis of assembling of time retarded solutions for the Liouville equation with the modified Bogolubov's condition of weakening correlations between particles. According to the Zubarev's nonequilibrium statistical operator method,^(24, 25) the total nonequilibrium distribution function $q(x^N; t)$ for all N particles of a system satisfies the following asymptotic condition:

$$\lim_{t \rightarrow -\infty} \exp(iL_N t_0)(q(x^N, t_0) - q_q(x^N, t_0)) = 0 \quad (2.1)$$

Here, $i = \sqrt{-1}$, and the limit $t_0 \rightarrow -\infty$ is taken after thermodynamic limiting transition $N \rightarrow \infty$, $V \rightarrow \infty$, $N/V \rightarrow \text{const}$; L_N is the Liouville operator:

$$L_N = \sum_{j=1}^N L(j) + \frac{1}{2} \sum_{j=1}^N \sum_{\substack{k=1 \\ j \neq k}}^N L(j, k)$$

$$L(j) = -i \frac{\mathbf{p}_j}{2m} \frac{\partial}{\partial \mathbf{r}_j} \quad (2.2)$$

$$L(j, k) = i \frac{\partial \Phi(|\mathbf{r}_{jk}|)}{\partial \mathbf{r}_{jk}} \left(\frac{\partial}{\partial \mathbf{p}_j} - \frac{\partial}{\partial \mathbf{p}_k} \right)$$

Φ_{jk} is the interaction energy between two particles j and k ; $x_j = \{\mathbf{r}, \mathbf{p}\}$ is the set of phase variables (coordinates and momenta).

Let us consider time scales when details of initial state $\varrho(x^N, t_0)$ become not so important, i.e., when $t \gg t_0$. Then, to avoid the dependency on t_0 let us average the formal solution to the Liouville equation $\varrho(x^N; t) = e^{-iL(t-t_0)}\varrho(x^N; t_0)$ with respect to initial times in the range $t_0 \div t$ and make the limiting transition $t_0 - t \rightarrow \infty$,^(24, 25)

$$\varrho(x^N; t) = \varepsilon \int_{-\infty}^0 dt' e^{\varepsilon t'} e^{iL_N t'} \varrho_q(x^N; t + t') \quad (2.3)$$

where ε tends to $+0$ after the thermodynamic limiting transition. The quasi-equilibrium distribution function $\varrho_q(x^N; t)$ is determined from the condition of maximum for informational entropy at fixed values of the single particle distribution function $f_1(x_1; t)$ and average density of the interaction energy $\langle \mathcal{E}_{\text{int}}(\mathbf{r}) \rangle^t$, $\langle \dots \rangle^t = \int d\Gamma_N \dots \varrho(x^N; t)$, $d\Gamma_N = (dx)^N/N!$. This corresponds to taking into account correlations, related to the conservations laws of hydrodynamical variables for particles density $n(\mathbf{r}; t)$, momentum $j(\mathbf{r}; t)$ and full energy $\mathcal{E}(\mathbf{r}; t)$.⁽²³⁾

The boundary condition (2.1) is equivalent to the transition from the Liouville equation to a modified one:^(24, 25)

$$\left(\frac{\partial}{\partial t} + iL_N \right) \varrho(x^N, t) = -\varepsilon(\varrho(x^N, t) - \varrho_q(x^N, t)) \quad (2.4)$$

This equation contains a small source on the right-hand side, which destroys the invariance with respect to time inversion.

The choice of $\varrho_q(x^N; t)$ depends mainly on nonequilibrium state of the system under consideration. In the case of low density gases, where times of free motion are essentially greater than collision times, higher-order distribution functions of particles become depend on time via oneparticle distribution functions only.⁽⁵⁾ It does mean that an abbreviated description of nonequilibrium states is available, and the total nonequilibrium distribution function depends on time via $f_1(x; t)$. In such a case, the quasiequilibrium distribution function $\varrho_q(x^N; t)$ reads:⁽⁵⁾

$$\varrho_q(x^N; t) = \prod_{j=1}^N \frac{f_1(x_j; t)}{e} \quad (2.5)$$

where e is the natural logarithm base. Then, the Liouville equation with a small source (2.4) in view of (2.5) corresponds to the abbreviated description of time evolution of the system on a kinetic stage when only the oneparticle distribution function is considered as a slow variable. However, there are always additional quantities, which vary in time slowly, because they are locally conserved. In the case of an onecomponent system, the mass density $\rho(\mathbf{r}; t)$, momentum $\mathbf{j}(\mathbf{r}; t)$ and total energy $\mathcal{E}(\mathbf{r}; t)$ belong to those quantities. At long times they satisfy the generalized hydrodynamics equations. Generally speaking, the equation for $f_1(x; t)$ should be conjugated with these equations. For low density gases, such a conjugation can be made, in principle, with arbitrary precision in each order over density. In high density gases and liquids, when a small parameter is absent, the correlation times corresponding to hydrodynamic quantities become to be commensurable with characteristic times of varying of oneparticle distribution functions. Therefore, in dense gases and liquids the kinetics and hydrodynamics are closely connected between themselves and they should be considered simultaneously. That is why, manyparticle correlations, related to the local conservation laws of mass, momentum and total energy, can not be neglected.^(24, 25) The local conservation laws affect on kinetic processes, because of the interaction of selected particles group with other particles of a system. This interaction is especially important in the case of high densities, and it must be included into consideration. Then at the construction of kinetic equations for high densities, it is necessary to choose the abbreviated description of nonequilibrium system in such a form to satisfy the true dynamics of conserved quantities automatically. To this end, the densities of hydrodynamic variables should be included together with the oneparticle distribution function $f_1(x; t)$ into the set of parameters of the abbreviated description initially.^(1, 22, 23) The next phase functions correspond to densities of hydrodynamic variables $\rho(\mathbf{r}; t)$, $\mathbf{j}(\mathbf{r}; t)$ and $\mathcal{E}(\mathbf{r}; t)$:

$$\begin{aligned}\hat{\rho}(\mathbf{r}) &= \int d\mathbf{p} m \hat{n}_1(x) \\ \hat{\mathcal{E}}(\mathbf{r}) &= \int d\mathbf{p} \frac{p^2}{2m} \hat{n}_1(x) + 1/2 \int d\mathbf{r}' d\mathbf{p} d\mathbf{p}' \Phi(|\mathbf{r} - \mathbf{r}'|) \hat{n}_2(x, x') \\ \hat{\mathbf{j}}(\mathbf{r}) &= \int d\mathbf{p} \mathbf{p} \hat{n}_1(x)\end{aligned}\quad (2.6)$$

where $\hat{n}_1(x)$ and $\hat{n}_2(x, x')$ are the one- and two-particle microscopic phase densities by Klimontovich.⁽¹³⁾ Relations (2.6) show a distinctive role of potential interaction energy. In contrary to $\rho(\mathbf{r}; t) = \langle \hat{\rho}(\mathbf{r}) \rangle^t$ and $\mathbf{j}(\mathbf{r}; t) = \langle \hat{\mathbf{j}}(\mathbf{r}) \rangle^t$, nonequilibrium values of the total energy $\mathcal{E}(\mathbf{r}; t) = \langle \hat{\mathcal{E}}(\mathbf{r}) \rangle^t$ can not be expressed via oneparticle distribution function $f_1(x; t) = \langle \hat{n}_1(x) \rangle^t$ exclusively, because to evaluate a potential part of $\mathcal{E}_{\text{int}}(\mathbf{r}; t) = \langle \hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) \rangle^t$ it is necessary to involve the twoparticle distribution function $f_2(x, x'; t) = \langle \hat{n}_2(x, x') \rangle^t$. Here

$$\hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) = \frac{1}{2} \int d\mathbf{r}' d\mathbf{p} d\mathbf{p}' \Phi(|\mathbf{r} - \mathbf{r}'|) \hat{n}_2(x, x') \quad (2.7)$$

is the density of potential energy of interaction. The next conclusion can be formulated as follows. If oneparticle distribution function $f_1(x; t)$ is chosen as a parameter of the abbreviated description, then the density of interaction energy (2.7) can be considered as an additional independent parameter. One can find the quasiequilibrium distribution function $\varrho_q(x^N; t)$ from the condition of extremum for the functional of informational entropy $S_{\text{int}}(t) = \int d\Gamma_N \varrho(x^N; t) \ln \varrho(x^N; t)$ at fixed average values of $\langle \hat{n}_1(x) \rangle^t = f_1(x; t)$, $\langle \hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) \rangle^t = \mathcal{E}_{\text{int}}(\mathbf{r}; t)$ including the normalization condition for $\varrho_q(x^N; t)$. After simple transformations the desired relation for the quasiequilibrium distribution function reads

$$\varrho_q(x^N; t) = \exp \left\{ -\Phi(t) - \int d\mathbf{r} \beta(\mathbf{r}; t) \hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) - \int dx a(x; t) \hat{n}_1(x) \right\} \quad (2.8)$$

$$\Phi(t) = \ln \int d\Gamma_N \exp \left\{ - \int d\mathbf{r} \beta(\mathbf{r}; t) \hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) - \int dx a(x; t) \hat{n}_1(x) \right\} \quad (2.9)$$

where $\Phi(t)$, $\beta(\mathbf{r}; t)$, $a(x; t)$ are the Lagrange multipliers. $\Phi(t)$ is the Massieu–Planck functional, which is determined from the condition of normalization for $\varrho_q(x^N; t)$: $\int d\Gamma_N \varrho_q(x^N; t) = 1$. Relation (2.8) for $\varrho_q(x^N; t)$

has been obtained for the first time in ref. 22. To determine the physical meaning of parameters $\beta(\mathbf{r}; t)$ and $a(x; t)$ let us rewrite $\varrho_q(x^N; t)$ (2.8) in the form

$$\varrho_q(x^N; t) = \exp \left\{ -\Phi(t) - \int d\mathbf{r} \beta(\mathbf{r}; t) \hat{\mathcal{E}}'(\mathbf{r}) - \int dx a(x; t) \hat{n}_1(x) \right\} \quad (2.10)$$

where $\hat{\mathcal{E}}'(\mathbf{r})$ is the density of total energy in a reference frame, which runs together a system element with the mass velocity $\mathbf{V}(\mathbf{r}; t)$:^(23, 24)

$$\hat{\mathcal{E}}'(\mathbf{r}) = \hat{\mathcal{E}}(\mathbf{r}) - \mathbf{V}(\mathbf{r}; t) \hat{\mathbf{j}}(\mathbf{r}) + \frac{m}{2} V^2(\mathbf{r}; t) \hat{n}(\mathbf{r}) \quad (2.11)$$

Here $\hat{n}(\mathbf{r}) = \int d\mathbf{p} \hat{n}_1(x)$ is the density of particles number. Parameters $\beta(\mathbf{r}; t)$ and $a(x; t)$ in (2.10) are defined from conditions of self-consistency, namely, the equality of quasiaveraging values $\langle \hat{n}_1(x) \rangle'_q$ and $\langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_q$ to their real average values $\langle \hat{n}_1(x) \rangle^t$, $\langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle^t$:

$$\begin{aligned} \langle \hat{n}_1(x) \rangle'_q &= \langle \hat{n}_1(x) \rangle^t = f_1(x; t) \\ \langle \hat{\mathcal{E}}'_1(\mathbf{r}) \rangle'_q &= \langle \hat{\mathcal{E}}'_1(\mathbf{r}) \rangle^t \end{aligned} \quad (2.12)$$

Here $\langle \dots \rangle'_q = \int d\Gamma_N \dots \varrho_q(x^N; t)$. In these transformations parameters $a'(x; t)$ and $a(x; t)$ are connected by the relation

$$a'(x; t) = a(x; t) - \beta(\mathbf{r}, t) \left\{ \frac{p^2}{2m} - \mathbf{V}(\mathbf{r}; t) \mathbf{p} + \frac{m}{2} V^2(\mathbf{r}; t) \right\}$$

In the case, when conditions (2.12) take place, one can obtain some relations, taking into account the self-consistency conditions and varying the modified Massieu–Planck functional

$$\Phi(t) = \ln \int d\Gamma_N \exp \left\{ - \int d\mathbf{r} \beta(\mathbf{r}; t) \hat{\mathcal{E}}'(\mathbf{r}) - \int dx a'(x; t) \hat{n}_1(x) \right\}$$

with respect to parameters $\beta(\mathbf{r}; t)$ and $a'(x; t)$

$$\begin{aligned} \frac{\delta \Phi(t)}{\delta \beta(\mathbf{r}; t)} &= - \langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_q = - \langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle^t \\ \frac{\delta \Phi(t)}{\delta a(x; t)} &= - \langle \hat{n}_1(x) \rangle'_q = - \langle \hat{n}_1(x) \rangle^t = -f_1(x; t) \end{aligned} \quad (2.13)$$

It means that the parameter $\beta(\mathbf{r}; t)$ is conjugated to the average energy in an accompanying reference frame, $a'(x; t)$ is conjugated to the nonequilibrium oneparticle distribution function $f_1(x; t)$. To determine the physical meaning of these parameters let us define entropy of the system taking into account the selfconsistency conditions (2.12)

$$\begin{aligned} S(t) &= -\langle \ln \varrho_q(x^N; t) \rangle'_q \\ &= \Phi(t) + \int d\mathbf{r} \beta(\mathbf{r}; t) \langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_t + \int dx a'(x; t) \langle \hat{n}_1(x) \rangle'_t \end{aligned} \quad (2.14)$$

Taking functional derivatives of $S(t)$ (2.14) with respect to $\langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_t$ and $\langle \hat{n}_1(x) \rangle'_t$ at fixed corresponding averaged values gives the following thermodynamic relations:

$$\frac{\delta S(t)}{\delta \langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_t} = \beta(\mathbf{r}; t), \quad \frac{\delta S(t)}{\delta f_1(x; t)} = a'(x; t) \quad (2.15)$$

Hence, $\beta(\mathbf{r}; t)$ is an analogue of local inverse temperature.

In general case, when kinetic and hydrodynamic processes are considered simultaneously, the quasiequilibrium distribution function (2.10) or (2.8) can be rewritten in somewhat other form. This form is more convenient for comparison with $\varrho_q(x^N; t)$ (2.5), obtained in the usual way,⁽⁵⁾ when $f_1(x; t)$ is only the parameter of the abbreviated description. First of all, let us note that one can include the parameter $\Phi(t)$ in (2.8) into parameter $a(x; t)$ as a term, which does not depend on x . Parameter $a(x; t)$ in $\varrho_q(x^N; t)$ can be excluded with the help of the self-consistency condition $\langle \hat{n}_1(x) \rangle'_q = \langle \hat{n}(x) \rangle'_t = f_1(x; t)$. Reducing of $\varrho_q(x^N; t)$ results in

$$\varrho_q(x^N; t) = \exp\{-U_N(\mathbf{r}^N; t)\} \prod_{l=1}^N \frac{f_1(x_l; t)}{u(\mathbf{r}_l; t)} \quad (2.16)$$

where functions $u(\mathbf{r}_l; t)$ are obtained from the relations

$$\begin{aligned} u(\mathbf{r}_l; t) &= \int \frac{d\mathbf{r}^{N-1}}{(N-1)!} \exp\{-U_N(\mathbf{r}, \mathbf{r}^{N-1}; t)\} \prod_{l=2}^N \frac{n(\mathbf{r}_l; t)}{u(\mathbf{r}_l; t)} \\ U_N(\mathbf{r}^N; t) &= U_N(\mathbf{r}_1, \dots, \mathbf{r}_N) = \frac{1}{2} \sum_{l \neq j=1}^N \Phi(|\mathbf{r}_l - \mathbf{r}_j|) \beta(\mathbf{r}_j; t) \end{aligned} \quad (2.17)$$

$n(\mathbf{r}; t) = \langle \hat{n}(\mathbf{r}) \rangle'_t = \int d\mathbf{p} f_1(x; t)$ is the nonequilibrium particles concentration. In last expression (2.16), $U_N(\mathbf{r}; t)$ and $u(\mathbf{r}_l; t)$ depend explicitly and implicitly, respectively, on $n(\mathbf{r}; t)$ and $\beta(\mathbf{r}; t)$ (or $\langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle'_t$). To obtain the

ordinary Bogolubov scheme,⁽⁵⁾ it is necessary to put $U_N(\mathbf{r}; t) = 0$ in (2.16) and (2.17). Then, one can define $u = e$, and (2.16) transforms into the quasiequilibrium distribution (2.5) as it should be. In general case $u(\mathbf{r}; t)$ is a functional of nonequilibrium density of particles number $\langle \hat{n}(\mathbf{r}) \rangle^t$ and $\beta(\mathbf{r}; t)$, which is an analogue of local inverse temperature. Nevertheless, one should handle with care with this analogy as far as definition (2.16) can describe states which are far from local equilibrium. In particular, $f_1(x; t)$ can distinguish considerably from the local Maxwellian distribution. The entropy expression (2.14) can be transformed accordingly to the structure of the quasiequilibrium distribution function (2.16)

$$S(t) = \int d\mathbf{r} \beta(\mathbf{r}; t) \langle \hat{\mathcal{E}}_{\text{int}}(\mathbf{r}) \rangle^t - \int dx f_1(x; t) \ln \frac{f_1(x; t)}{u(\mathbf{r}; t)} \quad (2.18)$$

Here “potential” and “kinetic” parts are separated. In the case of low density gases, the influence of the potential energy can be neglected and $u(\mathbf{r}; t) = e$. Then the expression (2.18) tends to the usual Boltzmann entropy.

3. THE MODIFIED GROUP EXPANSIONS

Integrating equation (2.4) over the phase space of $(N-s)$ particles, we obtain the equation chain for the s -particle nonequilibrium distribution function $f_s(x^s; t) = \int d\Gamma_{N-s} \varrho(x^N; t)$:^(1, 22)

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + iL_s \right) f_s(x^s; t) + \sum_{j=1}^s \int dx_{s+1} iL(j, s+1) f_{s+1}(x^{s+1}; t) \\ & = -\varepsilon \left(f_s(x^s; t) - g_s(\mathbf{r}^s; t) \prod_{j=1}^s f_1(x_j; t) \right) \end{aligned} \quad (3.1)$$

where $g_s(\mathbf{r}^s; t) = \int d\Gamma_{N-s} d\mathbf{p}^s \varrho_q(x^N, t)$ is the quasiequilibrium s -particle coordinate distribution function, which depends on $n(\mathbf{r}; t)$ and $\beta(\mathbf{r}; t)$ functionally. Due to the fact, that $g_1(\mathbf{r}_1; t) = 1$, the equation chain (3.1) is distinguished from the ordinary BBGKY hierarchy⁽¹⁸⁾ by the existence of sources in the right-hand parts of the equations beginning from the second one. Such sources take into account both one-particle and collective hydrodynamical effects.

Let us note, that equation chain (3.1) should be amplified by equations for spatial quasiequilibrium distribution functions $g_s(\mathbf{r}^s; t)$, which functionally depend on nonequilibrium density of particles number $n(\mathbf{r}; t)$ and “inversed” local temperature $\beta(\mathbf{r}; t)$. Specifically, it has been shown in ref. 26, that pair quasiequilibrium distribution function $g_2(\mathbf{r}_4, \mathbf{r}_2; t)$ is

connected with a pair quasiequilibrium correlation function: $h_2(\mathbf{r}_1, \mathbf{r}_2; t) = g_2(\mathbf{r}_1, \mathbf{r}_2; t) - 1$. On its turn, $h_2(\mathbf{r}_1, \mathbf{r}_2; t)$ satisfies the Ornstein-Zernike equation

$$h_2(\mathbf{r}_1, \mathbf{r}_2; t) = c_2(\mathbf{r}_1, \mathbf{r}_2; t) + \int d\mathbf{r}_3 c_2(\mathbf{r}_1, \mathbf{r}_3; t) h_2(\mathbf{r}_2, \mathbf{r}_3; t) n(\mathbf{r}_3; t)$$

where $c_2(\mathbf{r}_1, \mathbf{r}_2; t)$ is a direct quasiequilibrium correlation function.

To analyze the BBGKY hierarchy, we follow similarly to the papers by Zubarev and Novikov,^(5, 19-21) and earlier papers by⁽⁶⁾ and Cohen,^(4, 7) and cross from nonequilibrium distribution functions $f_s(x^s; t)$ to irreducible distribution ones $G_s(x^s; t)$, which may be introduced by equalities presented in refs. 5 and 6. In our case with some modifications we obtain

$$\begin{aligned} f_1(x_1; t) &= G_1(x_1; t) \\ f_2(x_1, x_2; t) &= G_2(x_1, x_2; t) + g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\ f_3(x_1, x_2, x_3; t) &= G_3(x_1, x_2, x_3; t) + \sum_P G_2(x_1, x_2; t) G_1(x_3; t) \\ &\quad + g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) G_1(x_1; t) G_1(x_2; t) G_1(x_3; t) \\ &\quad \vdots \end{aligned} \quad (3.2)$$

Here, the position-dependent quasiequilibrium distribution functions $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$, $g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t)$, $g_s(\mathbf{r}^s; t)$ are defined by relations of paper.⁽¹⁾ The modification of group expansions (3.2) consists in that a considerable part of the space time correlations is accumulated in quasiequilibrium functions $g_s(\mathbf{r}^s; t)$. If all $g_s(\mathbf{r}^s; t) = 1$ for $s = 2, 3, \dots$ these group expansions coincide with those of paper ref. 5. As far as each line in (3.2) brings exactly for new functions $G_s(\mathbf{r}^s; t)$, $s = 1, 2, 3, \dots$, the corresponding equations can be solved with respect to irreducible distribution functions and we may write the following:

$$\begin{aligned} G_1(x_1; t) &= f_1(x_1; t) \\ G_2(x_1, x_2; t) &= f_2(x_1, x_2; t) - g_2(\mathbf{r}_1, \mathbf{r}_2; t) f_1(x_1; t) f_1(x_2; t) \\ G_3(x_1, x_2, x_3; t) &= f_3(x_1, x_2, x_3; t) + \sum_P f_2(x_1, x_2; t) f_1(x_3; t) \\ &\quad - h_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) f_1(x_1; t) f_1(x_2; t) f_1(x_3; t) \\ &\quad \vdots \end{aligned} \quad (3.3)$$

In (3.2) and (3.3) the symbol \sum_P denotes the sum of all different permutations of coordinates for three and more particles,

$$\begin{aligned} h_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) &= g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) - g_2(\mathbf{r}_1, \mathbf{r}_2; t) - g_2(\mathbf{r}_1, \mathbf{r}_3; t) - g_2(\mathbf{r}_2, \mathbf{r}_3; t) \\ &\equiv h'_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) - 2 \end{aligned} \quad (3.4)$$

where $h'_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t)$ the three-particle quasiequilibrium correlation function. Now let us write the BBGKY hierarchy^(1, 2) for irreducible distribution functions $G_s(x^s; t)$, namely, the first two equations,

$$\begin{aligned} \left(\frac{\partial}{\partial t} + iL(1) \right) G_1(x_1; t) + \int dx_2 iL(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\ + \int dx_2 iL(1, 2) G_2(x_1, x_2; t) = 0 \end{aligned} \quad (3.5)$$

Differentiating the relation for $G_2(x_1, x_2; t)$ in (3.3) with respect to time and using the second equation from the BBGKY hierarchy for the function $f_2(x_1, x_2; t)$, we can get for the pair irreducible distribution function $G_2(x_1, x_2; t)$ the equation, which reads

$$\begin{aligned} \left(\frac{\partial}{\partial t} + iL_2 + \varepsilon \right) G_2(x_1, x_2; t) \\ = - \left(\frac{\partial}{\partial t} + iL_2 \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\ - \int dx_3 \{ iL(1, 3) + iL(2, 3) \} \left\{ G_3(x_1, x_2, x_3; t) \right. \\ + \sum_P G_2(x_1, x_2; t) G_1(x_3; t) \\ \left. + g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) G_1(x_1; t) G_1(x_2; t) G_1(x_3; t) \right\} \end{aligned} \quad (3.6)$$

In a similar way, we can obtain other equations for the three-particle irreducible function $G_3(x_1, x_2, x_3; t)$ and higher $G_s(x^s; t)$ ones. One remembers now, that appearing the quasiequilibrium distribution functions $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$, $g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t)$, $g_s(\mathbf{r}; t)$ in the hierarchy is closely connected with the fact that the boundary conditions for solutions of the Liouville equation take into consideration both the nonequilibrium character of one-particle distribution function and local conservation laws, that corresponds to a consistent description of kinetics and hydrodynamics of the system.^(1, 22) Since

in the present paper we analyze two first equations (3.5) and (3.6) only, we will not write down next ones. It is important to note, that if we put formally $g_s(\mathbf{r}^s; t) \equiv 1$ for all $s=2, 3, \dots$ in (3.5) and (3.6), we come to the first two equations of the BBGKY hierarchy for irreducible distribution functions $G_1(x_1; t)$ and $G_2(x_1, x_2; t)$, which were obtained in the paper ref. 5 by Zubarev and Novikov. The first term in the right-hand side of (3.6) is a peculiarity of (3.5) and (3.6) equations system. This is the term with time derivative of the pair quasiequilibrium distribution function $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$. As it was shown in refs. 1, 22, and 23, the pair quasiequilibrium distribution function is a functional of local values of the temperature $\beta(\mathbf{r}; t)$ and the mean particle density $n(\mathbf{r}; t)$. Thus, time derivatives of $g_2(\mathbf{r}_1, \mathbf{r}_2 | \beta(t), n(t))$ will be conformed to $\beta(\mathbf{r}; t)$ and $n(\mathbf{r}; t)$. These quantities, in its turn, according to the self-consistency conditions,⁽²³⁾ will be expressed via the average energy value $\langle \hat{\mathcal{E}}'(\mathbf{r}) \rangle^t$ in an accompanying reference frame and via $\langle \hat{n}(\mathbf{r}) \rangle^t$, which constitute a basis of the hydrodynamical description of nonequilibrium state of the system.

If we neglect the term in the right-hand side of equation (3.6), which takes into account ternary correlations between particles, we shall obtain from (3.6) the equation in the so-called "pair" collisions approximation with the following form:

$$\left(\frac{\partial}{\partial t} + iL_2 + \varepsilon \right) G_2(x_1, x_2; t) = - \left(\frac{\partial}{\partial t} + iL_2 \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \quad (3.7)$$

The formal solution to this equation reads

$$G_2(x_1, x_2; t) = - \int_{-\infty}^t dt' e^{(\varepsilon + iL_2)(t-t')} \left[\frac{\partial}{\partial t'} + iL_2 \right] g_2(\mathbf{r}_1, \mathbf{r}_2; t') \times G_1(x_1; t') G_1(x_2; t') \quad (3.8)$$

Inserting this solution into the first equation (3.5) we obtain the following kinetic equation for one-particle distribution function $f_1(x_1; t) = G_1(x_1; t)$:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + iL(1) \right) f_1(x_1; t) \\ &= - \int dx_2 iL(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t) f_1(x_1; t) f_1(x_2; t) \\ & \quad - \int dx_2 iL(1, 2) \int_{-\infty}^t dt' e^{(\varepsilon + iL_2)(t-t')} \left[\frac{\partial}{\partial t'} + iL_2 \right] g_2(\mathbf{r}_1, \mathbf{r}_2; t') \\ & \quad \times G_1(x_1; t') G_1(x_2; t') \end{aligned} \quad (3.9)$$

where the first term in the right-hand side is a generalization of the Vlasov mean field and corresponds to kinetic mean field theory, KMFT.^(27, 28) The kinetic equation (3.9) is completely equivalent to that of ref. 1, which is obtained in the “pair” collisions approximation.

Let us consider now the set of equations (3.5) and (3.6) in such an approximation that the threeparticle irreducible distribution function $G_3(x_1, x_2, x_3; t)$ and $h_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t)$ are neglected in the second equation (3.6). This is in the spirit of the polarization approximation introduced for obtaining the Lenard–Balescu kinetic equation for a homogeneous Coulomb plasma.⁽¹⁵⁾ Taking into account (3.4) as well as the relation $G_3(x_1, x_2, x_3; t) \equiv 0$ and similarly $h_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) \equiv 0$, we rewrite the equation (3.6) in the form:

$$\begin{aligned}
 & \left(\frac{\partial}{\partial t} + iL_2 + \varepsilon \right) G_2(x_1, x_2; t) \\
 &= - \left(\frac{\partial}{\partial t} + iL_2 \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\
 & \quad - \int dx_3 iL(1, 3) \{ G_2(x_1, x_2; t) G_1(x_3; t) + G_2(x_2, x_3; t) G_1(x_1; t) \} \\
 & \quad - \int dx_3 iL(2, 3) \{ G_2(x_1, x_2; t) G_1(x_3; t) + G_2(x_1, x_3; t) G_1(x_2; t) \} \\
 & \quad - \int dx_3 \{ iL(1, 3) + iL(2, 3) \} \{ g_2(\mathbf{r}_1, \mathbf{r}_2; t) + g_2(\mathbf{r}_1, \mathbf{r}_3; t) + g_2(\mathbf{r}_2, \mathbf{r}_3; t) \} \\
 & \quad \times G_1(x_1; t) G_1(x_2; t) G_1(x_3; t) \tag{3.10}
 \end{aligned}$$

Next, let us introduce the operator, which can be obtained by variation of the Vlasov collision integral near a nonequilibrium distribution $G_1(x_1; t)$:

$$\begin{aligned}
 \delta \left(\int dx_3 iL(1, 3) G_1(x_3; t) G_1(x_1; t) \right) &= \int dx_3 iL(1, 3) G_1(x_3; t) G_1(x_1; t) \\
 &= \mathcal{L}(x_1; t) \delta G_1(x_1; t) \tag{3.11}
 \end{aligned}$$

Then one represents the equation (3.10) with the help of operator $\mathcal{L}(x_1; t)$ in the form

$$\begin{aligned}
 & \left(\frac{\partial}{\partial t} + iL_2 + \mathcal{L}(x_1, x_2; t) + \varepsilon \right) G_2(x_1, x_2; t) \\
 &= - \left(\frac{\partial}{\partial t} + iL_2 + \mathcal{L}(x_1, x_2; t) \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \tag{3.12}
 \end{aligned}$$

from which the formal solution for the irreducible two-particle distribution function reads

$$G_2(x_1, x_2; t) = - \int_{-\infty}^t dt' e^{\epsilon(t'-t)} U(t, t') \left(\frac{\partial}{\partial t'} + iL_2 + \mathcal{L}(x_1, x_2; t') \right) \times g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_1; t') G_1(x_2; t') \quad (3.13)$$

and $U(t, t')$ is the evolution operator,

$$U(t, t') = \exp_+ \left\{ - \int_{t'}^t dt'' (iL_2 + \mathcal{L}(x_1, x_2; t'')) \right\} \quad (3.14)$$

$$\mathcal{L}(x_1, x_2; t) = \mathcal{L}(x_1; t) + \mathcal{L}(x_2; t)$$

As a result, we obtain the expression for the irreducible quasiequilibrium two-particle distribution function $G_2(x_1, x_2; t)$ in the generalized polarization approximation. Inserting this expression (3.13) into the first equation of the chain (3.5) yields:

$$\left(\frac{\partial}{\partial t} + iL(1) \right) G_1(x_1; t) + \int dx_2 iL(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) = \int dx_2 \int_{-\infty}^t dt' e^{\epsilon(t'-t)} iL(1, 2) U(t, t') \left(\frac{\partial}{\partial t'} + iL_2 + \mathcal{L}(x_1, x_2; t') \right) \times g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_1; t') G_1(x_2; t') \quad (3.15)$$

This is the kinetic equation for the nonequilibrium oneparticle distribution function with the nonMarkovian collision integral in the generalized polarization approximation. It is necessary to note, that the presence of the Vlasov's operator $\mathcal{L}(x_1, x_2; t)$ in the collision integral (3.15) indicates about taking into consideration collective effects. An analysis of the collision integral (3.15) in the general case is rather a complicated problem. But it is obvious, that the collision integral in (3.15), or the expression for $G_2(x_1, x_2; t)$ in (3.13) may be much simplified for every physical model of a particle system, or for each nonequilibrium state of the collision integral in (3.15). To show this, we shall consider two concrete cases: a hard spheres model and a Coulomb plasma.

4. HARD SPHERES MODELS IN THE POLARIZATION APPROXIMATION

In this section we shall perform the investigation of kinetic processes for hard spheres model in approximations, which are higher than "pair"

collisions one. We take into account the character of model parameters and the results of the previous section of this article and papers of refs. 14, 15, and 29. This investigation is convenient to carry out on the basis of equation chain (3.5), (3.6) at formal substitution of a potential part of the Liouville operator $iL(1, 2)$ (refs. 1 and 2) by the Enskog collision operator $\hat{T}(1, 2)$.^(1, 2) In this case, equations (3.5) and (3.6) have the form

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + iL(1) \right) G_1(x_1; t) + \int dx_2 \hat{T}(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\ & + \int dx_2 \hat{T}(1, 2) G_2(x_1, x_2; t) = 0 \end{aligned} \quad (4.1)$$

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + iL_2^0 + \hat{T}(1, 2) + \varepsilon \right) G_2(x_1, x_2; t) \\ & = - \left(\frac{\partial}{\partial t} + iL_2^0 + \hat{T}(1, 2) \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \\ & - \int dx_3 \{ \hat{T}(1, 3) + \hat{T}(2, 3) \} \\ & \times \left\{ G_3(x_1, x_2, x_3; t) + \sum_P G_2(x_1, x_2; t) G_1(x_3; t) \right. \\ & \left. + g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) G_1(x_1; t) G_1(x_2; t) G_1(x_3; t) \right\} \end{aligned} \quad (4.2)$$

Further, we will consider the same approximations concerning to equation (4.2), in which $G_3(x_1, x_2, x_3; t)$ and $h_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t)$ are neglected. Then, if we introduce similarly to (3.11) the Boltzmann–Enskog collision operator $C(x_1; t)$, the equation (4.2) could be rewritten in the next form:

$$\delta \int dx_3 \hat{T}(1, 3) G_1(x_1; t) G_1(x_3; t) = C(x_1; t) \delta G_1(x_1; t) \quad (4.3)$$

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2; t) + \varepsilon \right) G_2(x_1, x_2; t) \\ & = - \left(\frac{\partial}{\partial t} + iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2; t) \right) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) \end{aligned} \quad (4.4)$$

Hence it appears, that the formal solution to $G_2(x_1, x_2; t)$ reads

$$G_2 \hat{n} = - \int_{-\infty}^0 dt' e^{e(t'-t)} U_{\text{hs}}(t, t') \left\{ \frac{\partial}{\partial t'} + iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2; t') \right\} \\ \times g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_1; t') G_1(x_2; t') \quad (4.5)$$

where $U_{\text{hs}}(t, t')$ is the evolution operator for the system of hard spheres:

$$U_{\text{hs}}(t, t') = \exp_+ \left\{ - \int_{t'}^t dt'' [iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2; t'')] \right\} \quad (4.6)$$

$$C(x_1, x_2; t) = C(x_1; t) + C(x_2; t)$$

Now let us put (4.5) into the first equation (4.1). Then the resulting equation looks:

$$\left(\frac{\partial}{\partial t} + iL(1) \right) G_1(x_1; t) \\ = \int dx_2 \hat{T}(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t) G_1(x_1; t) G_1(x_2; t) - \int dx_2 \hat{T}(1, 2) \\ \times \int_{-\infty}^0 dt' e^{e(t'-t)} U_{\text{hs}}(t, t') \left\{ \frac{\partial}{\partial t'} + iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2; t') \right\} \\ \times g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_1; t') G_1(x_2; t') \quad (4.7)$$

This equation can be called as a generalized kinetic equation for the non-equilibrium one-particle distribution function of hard spheres with the non-Markovian collision integral in the generalized polarization approximation. The first term in the right-hand side of this equation is the collision integral from the revised Enskog theory.⁽³⁰⁾ Neglecting time retardation effects and assuming that the operator $C(x_1, x_2; t)$ does not depend on time when

$$G_1(x_1; t) = f_0(\mathbf{p}) = \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left\{ - \frac{p^2}{2mkT} \right\}$$

is the local equilibrium Maxwell distribution function, the next term can be rewritten in a simplified form,

$$I_{\mathbf{R}}(x_1; t) = - \int_{-\infty}^t dt' e^{e(t'-t)} R_0(x_1; t, t') G_1(x_1; t') \\ - \int_{-\infty}^t dt' e^{e(t'-t)} R_0(x_1; t, t') G_1(x_1; t') \quad (4.8)$$

where

$$R_0(x_1; t, t') = \int dx_2 \hat{T}(1, 2) e^{\{(t'-t)(iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2))\}} \\ \times [iL_2^0 + C(x_1, x_2)] g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_2; t') \quad (4.9)$$

$$R_1(x_1; t, t') = \int dx_2 \hat{T}(1, 2) e^{\{(t'-t)(iL_2^0 + \hat{T}(1, 2) + C(x_1, x_2))\}} \\ \times \hat{T}(1, 2) g_2(\mathbf{r}_1, \mathbf{r}_2; t') G_1(x_2; t') \quad (4.10)$$

$R_1(x_1; t, t')$ is the generalized ring operator. The kinetic equation (4.7) together with (4.9), (4.10) is the generalization of the kinetic equation for a system of hard spheres, which has been obtained by Bogolubov in refs. 15 and 29. It coincides with that, when the quasiequilibrium pair distribution function of hard spheres is set formally to be unity.

5. COULOMB PLASMA IN THE POLARIZATION APPROXIMATION

Here we shall study an electron gas, which is contained into a homogeneous positively charged equilibrating background. This background can be created, for example, by hard motionless ions. Then, electrons interact according to the Coulomb law:

$$\Phi(|\mathbf{r}_{12}|) = \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{e^2}{|\mathbf{r}_{12}|}$$

the Fourier transform of which exists in the form of a real function $\phi(|\mathbf{k}|)$:

$$\frac{e^2}{r_{12}} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{4\pi e^2}{k^2} e^{i\mathbf{k} \cdot \mathbf{r}_{12}}, \quad \Phi(k) = \frac{4\pi e^2}{k^2} \quad (5.1)$$

here \mathbf{k} is a wavevector, e is the electron charge. Let us consider equation chain (3.5), (3.10) in the homogeneous case, when $G_1(x_1; t) = G_1(\mathbf{p}_1; t)$ and pair distribution functions depend on $|\mathbf{r}_{12}|$. Following the Bogolubov method,^(15, 18) we shall suppose that the one-particle distribution function $G_1(\mathbf{p}_1; t)$ is calculated in the “zeroth” order on interaction constant q , pair distribution functions $G_2(\mathbf{r}_{12}, \mathbf{p}_1, \mathbf{p}_2; t)$ and $g_2(\mathbf{r}_{12}; t)$ as the first order on q , and $G_3(x_1, x_2, x_3; t)$, $g_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; t) \sim q^2$, where $q = e^2/r_d \Theta$, $r_d = \sqrt{\Theta/4\pi e^2 n}$ is the Debye radius, $n = N/V$, $\Theta = k_B T$, k_B is the Boltzmann constant, T is thermodynamic temperature. Therefore, to obtain an equation for $G_2(\mathbf{r}_{12}, \mathbf{p}_1, \mathbf{p}_2; t)$ in the first approximation on interaction constant

q without time retardment effects, it is necessary to retain all integral terms, but omit all the others. In this case, using the Fourier transform with respect to spatial coordinates for a homogeneous Coulomb electron gas, the set of equations (3.5), (3.10) yields:

$$\begin{aligned} \frac{\partial}{\partial t} G_1(\mathbf{p}_1; t) &= -\frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{k} d\mathbf{p}_2 i\Phi(|\mathbf{k}|) g_2(\mathbf{k}; t) G_1(\mathbf{p}_1; t) G_1(\mathbf{p}_2; t) \\ &\quad -\frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{k} d\mathbf{p}_2 i\Phi(|\mathbf{k}|) G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t) \end{aligned}$$

or

$$\begin{aligned} \frac{\partial}{\partial t} G_1(\mathbf{p}_1; t) &= \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) \int d\mathbf{k} \mathbf{k} \Phi(|\mathbf{k}|) \Im g_2(\mathbf{k}; t) \\ &\quad + \frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{k} \mathbf{k} \Phi(|\mathbf{k}|) \Im G_2(\mathbf{k}, \mathbf{p}_1; t) \end{aligned} \quad (5.2)$$

equation for $G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t)$:

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + i\mathbf{k} \frac{\mathbf{p}_{12}}{m} + \varepsilon \right) G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t) \\ &= i\mathbf{k} \Phi(|\mathbf{k}|) \left\{ \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) \int d\mathbf{p}_3 G_2(\mathbf{k}, \mathbf{p}_2, \mathbf{p}_3; t) \right. \\ &\quad \left. - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) \int d\mathbf{p}_3 G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_3; t) \right\} \\ &\quad + i\mathbf{k} \Phi(|\mathbf{k}|) \left\{ \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) g_2(-\mathbf{k}; t) G_1(\mathbf{p}_2; t) \right. \\ &\quad \left. - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) g_2(\mathbf{k}; t) G_1(\mathbf{p}_1; t) \right\} \end{aligned} \quad (5.3)$$

$\varepsilon \rightarrow +0$, and $G_2(\mathbf{k}, \mathbf{p}_1; t) = \int d\mathbf{p}_2 G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t)$; $\Im g_2(\mathbf{k}; t)$, $\Im G_2(x_1, x_2; t)$ are imaginary parts of the corresponding distribution functions. The following properties should be noted:

$$\begin{aligned} G_2(-\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t) &= G_2^*(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t) \\ g_2(-\mathbf{k}; t) &= g_2^*(\mathbf{k}; t) \end{aligned}$$

The solution to (5.3), neglecting time retardment effects, reads:

$$\begin{aligned}
 & G_2(\mathbf{k}, \mathbf{p}_1, \mathbf{p}_2; t) \\
 &= \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} \left\{ \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) G_2(-\mathbf{k}, \mathbf{p}_2; t) - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) G_2(\mathbf{k}, \mathbf{p}_1; t) \right\} \\
 &+ \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} \left\{ \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) g_2(-\mathbf{k}, \mathbf{p}_2; t) - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) g_2(\mathbf{k}, \mathbf{p}_1; t) \right\}
 \end{aligned} \tag{5.4}$$

It should be noticed also that equation (5.2) contains an imaginary part of the pair irreducible nonequilibrium distribution function, to be integrated with respect to momentum of the second particle. Now one integrates equation (5.4) over all values of momentum \mathbf{p}_2 and defines in such a way some function $G_2(\mathbf{k}, \mathbf{p}_1; t)$:

$$\begin{aligned}
 & \left(1 + \int d\mathbf{p}_2 \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) \right) G_2(\mathbf{k}, \mathbf{p}_1; t) \\
 &= \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) \int d\mathbf{p}_2 \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} G_2(-\mathbf{k}, \mathbf{p}_2; t) \\
 &+ \int d\mathbf{p}_2 \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} \left\{ \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) g_2(-\mathbf{k}; t) G_1(\mathbf{p}_2; t) \right. \\
 &\left. - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) g_2(\mathbf{k}; t) G_1(\mathbf{p}_1; t) \right\}
 \end{aligned} \tag{5.5}$$

Further, we should exclude from (5.5) the term with $G_2(-\mathbf{k}, \mathbf{p}_2; t)$. To do this, we follow Lenard^(33, 34) and integrate the equation (5.5) over momentum component $\mathbf{p}_{1\perp}$, which is perpendicular to wavevector \mathbf{k} . Resulting expression then reads:

$$\begin{aligned}
 & [1 + \Phi(|\mathbf{k}|) \chi(\mathbf{k}, p_1; t)] G_2(\mathbf{k}, p_1; t) \\
 &= \frac{\partial}{\partial \mathbf{p}_1} G_1(p_1; t) \int d\mathbf{p}_2 \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} G_2(-\mathbf{k}, \mathbf{p}_2; t) \\
 &+ \int d\mathbf{p}_2 \frac{\mathbf{k}\Phi(|\mathbf{k}|)}{\mathbf{k} \cdot (\mathbf{p}_{12}/m) - i0} \left\{ \frac{\partial}{\partial p_1} G_1(p_1; t) g_2(-\mathbf{k}; t) G_1(\mathbf{p}_2; t) \right. \\
 &\left. - \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t) g_2(\mathbf{k}; t) G_1(p_1; t) \right\}
 \end{aligned} \tag{5.6}$$

Here the following conventional designations have been introduced:

$$\chi(\mathbf{k}, p_1; t) = \int d\mathbf{p}_2 \frac{\mathbf{k}}{k \cdot (p_{12}/m) - i0} \frac{\partial}{\partial \mathbf{p}_2} G_1(\mathbf{p}_2; t),$$

$$p_1 = \frac{\mathbf{p}_1 \cdot \mathbf{k}}{k}, \quad p_2 = \frac{\mathbf{p}_2 \cdot \mathbf{k}}{k}, \quad k = |\mathbf{k}|$$
(5.7)

$$G_1(p_1; t) = \int d\mathbf{p}_{1\perp} G_1(\mathbf{p}_1; t)$$

$$G_2(\mathbf{k}, p_1; t) = \int d\mathbf{p}_{1\perp} G_2(\mathbf{k}, \mathbf{p}_1; t)$$

Now we multiply both equations (5.5) and (5.6) on $(\partial/\partial p_1) G_1(p_1; t)$ and $(\partial/\partial p_1) G_1(\mathbf{p}_1; t)$, respectively, and subtract them:

$$(1 + \Phi(|\mathbf{k}|) \chi(\mathbf{k}, p_1; t)) \left[G_2(\mathbf{k}, \mathbf{p}_1; t) \frac{\partial}{\partial p_1} G_1(p_1; t) \right. \\ \left. - G_2(\mathbf{k}, p_1; t) \frac{\partial}{\partial p_1} G_1(\mathbf{p}_1; t) \right] \\ = \Phi(|\mathbf{k}|) \chi(\mathbf{k}, p_1; t) g_2(\mathbf{k}; t) \left[G_1(p_1; t) \frac{\partial}{\partial p_1} G_1(\mathbf{p}_1; t) \right. \\ \left. - G_1(\mathbf{p}_1; t) \frac{\partial}{\partial p_1} G_1(p_1; t) \right]$$
(5.8)

If we extract imaginary part of this equation, one can find unknown quantity $\Im m G_2(\mathbf{k}, \mathbf{p}_1; t)$ provided $\Im m G_2(\mathbf{k}, p_1; t) = 0$:⁽³³⁾

$$\frac{\partial}{\partial p_1} G_1(p_1; t) \Im m G_2(\mathbf{k}, \mathbf{p}_1; t) \\ = \frac{\Phi(|\mathbf{k}|) \Im m[\chi(\mathbf{k}, p_1; t) g_2(\mathbf{k}; t)]}{|1 + \Phi(|\mathbf{k}|) \chi(\mathbf{k}, p_1; t)|^2} \\ \times \left[G_1(p_1; t) \frac{\partial}{\partial p_1} G_1(\mathbf{p}_1; t) - G_1(\mathbf{p}_1; t) \frac{\partial}{\partial p_1} G_1(p_1; t) \right]$$
(5.9)

Since $\Im m \chi(\mathbf{k}, p_1; t) = -\pi(\partial/\partial p_1) G_1(p_1; t)$,^(33, 34) putting the expression for $\Im m G_2(\mathbf{k}, \mathbf{p}_1; t)$ into the equation (5.2) gives the generalized Bogolubov–Lenard–Balescu kinetic equation for an electron gas in an equilibrating background

$$\begin{aligned} \frac{\partial}{\partial t} G_1(\mathbf{p}_1; t) &= \frac{\partial}{\partial \mathbf{p}_1} G_1(\mathbf{p}_1; t) \int d\mathbf{k} \mathbf{k} \Phi(|\mathbf{k}|) \Im g_2(\mathbf{k}; t) \\ &+ \frac{\partial}{\partial \mathbf{p}_1} \int d\mathbf{p}_2 Q(\mathbf{p}_1, \mathbf{p}_2; t) \left[\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2} \right] G_1(\mathbf{p}_1; t) G_1(\mathbf{p}_2; t) \end{aligned} \quad (5.10)$$

where $Q(\mathbf{p}_1, \mathbf{p}_2; t)$ is a second rank tensor

$$Q(\mathbf{p}_1, \mathbf{p}_2; t) = -\pi \int d\mathbf{k} \frac{|\Phi(|\mathbf{k}|)|^2 \mathbf{k} \cdot \mathbf{k}}{|1 + \Phi(|\mathbf{k}|) \chi(\mathbf{k}, p_1; t)|^2} \Im g_2(\mathbf{k}; t) \delta(\mathbf{k} \cdot (\mathbf{p}_1 - \mathbf{p}_2)) \quad (5.11)$$

which coincides with $Q(\mathbf{p}_1, \mathbf{p}_2)$ (ref. 32) at $\Im g_2(\mathbf{k}; t) = 1$. In this case, the kinetic equation (5.10) transforms into the well-known Lenard–Balescu equation.^(32–34) Evidently, the generalized Bogolubov–Lenard–Balescu kinetic equation (5.10) climes the description of a dense electron gas, since in the both generalized mean field and generalized Bogolubov–Lenard–Balescu collision integrals, manyparticle correlations are treated by imaginary part of $g_2(\mathbf{k}; t)$. Nevertheless, the problem of divergence in the collision integral of equation (5.10) at small distances ($k \rightarrow \infty$) still remains. There are papers, where divergence of collision integrals is avoided with the help of special choice of differential cross section (quantum systems⁽³⁵⁾), or via combination of simpler collision integrals (classical systems⁽¹³⁾). These generalization for collision integrals are attractive by theirs simplicity and are usable for ideal plasma. But, contrary to the obtained by us Bogolubov–Lenard–Balescu kinetic equation, they do not work for nonideal plasma. In accordance to the proposed structure of collision integral

$$I_{\text{total}} = I_{\text{Boltzmann}} - I_{\text{Landau}} + I_{\text{Lenard–Balescu}}$$

the influence of particles interaction into plasma energy will be defined by correlation function $g_2(r)$. Its asymptotic is

$$\lim_{r \rightarrow \infty} \left[\underbrace{\exp \left\{ -\frac{e_a e_b}{rk_B T} \right\}}_{\text{Boltzmann}} - 1 + \underbrace{\frac{e_a e_b}{rk_B T}}_{\text{Landau}} - \underbrace{\frac{e_a e_b}{rk_B T} e^{-r/r_D}}_{\text{Lenard–Balescu}} \right] = \frac{1}{r^2} (!),$$

where r_D denotes the Debye radius

At other combinations one arrives at false expressions for thermodynamic functions.⁽¹³⁾ Dynamical screening, appearing in the obtained by us generalized Bogolubov–Lenard–Balescu collision integral, is free of these

discrepancies. Generally speaking, the problem of divergency could be solved within a frame of charged hard spheres model, combining the results of this section and previous one. But this step constitutes an intricate and complicated problem and needs a separate consideration.

Evidently, an investigation of the obtained kinetic equation is important in view of its solutions and studying transport Coefficients and time correlation functions for model systems.

6. CONCLUSION

In view of dense systems study, where the consideration of spatially interparticle correlations is important, the BBGKY hierarchy (3.5), (3.6) with the modified boundary conditions and group expansions has a quite good perspective. The kinetic equation (4.7)–(4.10) is a generalization of the Bogolubov one^(15, 29) for a system of hard spheres. Ernst and Dorfman⁽³¹⁾ had investigated collective modes in a nonhomogeneous gas and showed, that the solution of a dispersion equation for hydrodynamic modes leads to the nonanalytic frequency dependence on wavevector. This is connected with the fact, that the ring operator for nonhomogeneous systems at small wavenumbers has a term proportional to \sqrt{k} . Similar investigations of collective modes and time correlation functions in the hydrodynamic region have been carried out by Bogolubov.^(15, 29, 32) Nevertheless, it is necessary to carry out analogous investigations of hydrodynamic collective modes and time correlation functions on the basis of kinetic equation (4.7), taking into account (4.8)–(4.10), where some part of space correlations is considered in the pair quasiequilibrium function $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$. Obviously, these results may appear to be good for very dense gases, which could be described by a hard spheres model. An important factor is that in the kinetic equation (4.7)–(4.10) as well as in the generalized Bogolubov–Lenard–Balescu one, collective effects are taking into account both via Vlasov’s mean field and pair quasiequilibrium correlation function, which is a functional of nonequilibrium values of temperature and chemical potential.

Transferring the obtained results on quantum systems is not obvious. Such a procedure is rather complicated and needs additional investigations. Nevertheless, some steps in this way have been done already by our colleagues.^(36, 37)

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